An Optimal Solution of Fuzzy Transportation Problem

T. Leelavathy and K. Ganesan

Abstract - In this paper, we present a methodology for solving Fuzzy Transportation Problem (FTP) when all the cost coefficients are fuzzy numbers and all demands and supplies are crisp numbers. To obtain Initial Fuzzy Basic Feasible Solution (IFBFS) we use fuzzy version of Vogel's Approximation Method (FVAM) and then by using fuzzy version of Modified Distribution Method (FMODI), we obtain the fuzzy optimal solution for fuzzy transportation problem without converting to a classical transportation problem. Finally the feasibility of the proposed study is discussed with a numerical example.

Index Terms—Fuzzy transportation problem, Triangular fuzzy number, Fuzzy arithmetic, Fuzzy Optimal solution.

1. INTRODUCTION

The transportation problem is one of the earliest applications of linear programming problems. The objective function is to minimize total transportation costs and satisfy the destination requirements within the source availability [16]. Within a given time period each shipping source has a certain capacity and each destination has certain requirements with a given cost of shipping from the source to the destination. In order to solve a transportation problem the decision parameters of the problem must be fixed at crisp values. However, in real life situations, the information available is of imprecise nature and there is an inherent degree of vagueness or uncertainty present in the problem under consideration. In order to tackle this uncertainty the concept of fuzzy sets can be used as an important decision making tool. These imprecise data may be represented by fuzzy numbers.

The idea of fuzzy set was introduced by Zadeh in 1965. Bellmann and Zadeh [3] proposed the concept of decision making in fuzzy environment. Chanas et.al [5] developed a method for solving fuzzy transportation problems by applying the parametric programming technique using the Bellman–Zadeh criterion [3]. Chanas and Kuchta [4] proposed a method for solving a fuzzy transportation problem by converting the given problem to a bicriterial transportation problem with crisp objective function which provides only crisp solution to the given problem. Liu and Kao [16] proposed a new method for the solution of the fuzzy transportation problem by using the Zadeh's extension principle. Using parametric approach, Nagoorgani and Abdul Razak [12] obtained a fuzzy solution for a two stage Fuzzy Transportation problem with trapezoidal fuzzy numbers.

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Pandian and Natarajan [17] proposed a fuzzy zero point method to find the fuzzy optimal solution of fuzzy transportation problems. In general, the authors have transformed the given fuzzy transportation problem in to one or more crisp transportation problems and then obtained the crisp optimal solution. In this paper, we present a methodology for solving Fuzzy Transportation Problem (FTP) when all the cost coefficients are fuzzy numbers and all demands and supplies are crisp numbers, without converting to a classical transportation problem.

The rest of the paper is organized as follows: In section 2, the basic concepts of fuzzy numbers, definitions, membership function, the results of triangular fuzzy number and their arithmetic operations and the related results were discussed. In section 3, we define the Fuzzy Transportation Problem (FTP) with triangular fuzzy numbers and introduced related results. In section 4, we present Fuzzy Version of Vogel's Approximation Algorithm (FVAM) and fuzzy version of Modified distribution method (FMODI), to obtain fuzzy optimal solution to Fuzzy Transportation problem, without converting them to classical transportation problem. In section 5, a numerical example is provided to illustrate the efficiency of the proposed methodology.

2. PRELIMINARIES

Definition 2.1. A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number, if its membership function $\tilde{A} : R \to [0,1]$ has the following characteristics:

(i). $\tilde{A}$ is convex,

i.e.,$\tilde{A}\left\{\lambda x_1 + (1-\lambda)x_2\right\} \geq \min\left\{\tilde{A}(x_1),\ \tilde{A}(x_2)\right\}$,

$for\ all\ x_1, x_2 \in R\ and\ \lambda \in [0,1]$

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(ii). $\tilde{A}$ is normal, i.e., there exists an $x \in R$ such that $\tilde{A}(x)=1$.

(iii). $\tilde{A}$ is upper semi-continuous.

(iv). supp $(\tilde{A})$ is bounded in R.

Definition 2.2. A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3), where a_1, a_2, a_3 are real numbers and its membership function $\tilde{A}(x)$ is given below:

$$\tilde{A}(x) = \begin{cases} 0 & , \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1} & , \quad a_1 < x < a_2 \\ 1 & , \quad x = a_2 \\ \frac{a_3-x}{a_3-a_2} & , \quad a_2 < x < a_3 \\ 0 & , \quad x > a_3 \end{cases}$$

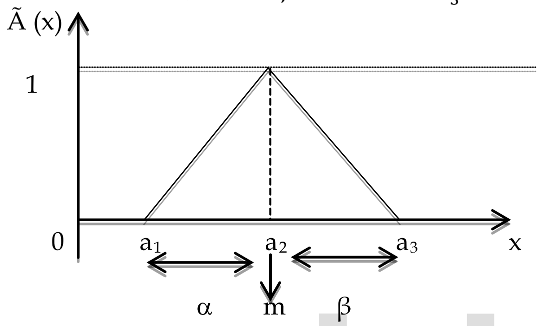

Fig.1.Triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3) = (\alpha, m, \beta)$

Also if $m=a_2$ represents the modal value or midpoint, $\alpha=(a_2-a_1)$ represents the left spread and $\beta=(a_3-a_2)$ represents the right spread of the triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\tilde{A}$ can be represented by the triplet $\tilde{A} = (\alpha, m, \beta)$.

Definition 2.3. A triangular fuzzy number $\tilde{A} \in F(R)$ can also be represented as a pair $\tilde{A} = (\underline{a}, \overline{a})$ of functions $\underline{a}(r)$ and $\overline{a}(r)$ for $0 \le r \le 1$ which satisfies the following requirements:

(i) $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

(ii) $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

(iii) $\underline{a}(r) \le \overline{a}(r)$, $0 \le r \le 1$.

Definition 2.4. For an arbitrary triangular fuzzy number $\tilde{A} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\frac{\underline{a}(1)+\overline{a}(1)}{2}\right)$ is said to be a location index number of $\tilde{A}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$, $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number $\tilde{A}=(a_1, a_2, a_3)$ can also be represented by $\tilde{A} = (a_0, a_*, a^*)$.

2.1. Ranking of Triangular Fuzzy Numbers

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. That is, for every $\tilde{A} = (a_1, a_2, a_3) \in F(R)$, the ranking function $\Re : F(R) \to R$ by graded mean is defined as $\Re(\tilde{A}) = \left(\frac{a_1 + 4a_2 + a_3}{6}\right)$.

For any two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ in F(R), we have the following comparison:

(i) $\tilde{A} \succcurlyeq \tilde{B}$ if and only if $\Re(\tilde{A}) \ge \Re(\tilde{B})$

(ii) $\tilde{A} \preccurlyeq \tilde{B}$ if and only if $\Re(\tilde{A}) \le \Re(\tilde{B})$

(iii) $\tilde{A} \approx \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$

A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ in F(R) is said to be positive, if and only if $\Re(\tilde{A}) > 0$ and is denoted by $\tilde{A} \succ 0$.

Two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ in F(R) are said to be equivalent if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$ and is denoted by $\tilde{A} \approx \tilde{B}$.

2.2. Arithmetic Operations on Triangular Fuzzy Numbers

Ming Ma et al. [18] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for $a,b \in L$ we define $a \vee b = \max\{a,b\}$ and $a \wedge b = \min\{a,b\}$.

For arbitrary triangular fuzzy numbers $\tilde{A} = (a_0, a_*, a^*)$ and $\tilde{B} = (b_0, b_*, b^*)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by

$$\begin{aligned} \tilde{A} * \tilde{B} &= (a_0 * b_0 , a_* \vee b_* , a^* \vee b^*) \\ &= (a_0 * b_0 , \max(a_*, b_*) , \max(a^*, b^*)) \end{aligned}$$

In particular for any two triangular fuzzy numbers $\tilde{A} = (a_0, a_*, a^*)$ and $\tilde{B} = (b_0, b_*, b^*)$, we define:

(i) Addition: $\tilde{A} + \tilde{B} = (a_0, a_*, a^*) + (b_0, b_*, b^*)$
$= (a_0 + b_0 , \max\{a_*, b_*\}, \max\{a^*, b^*\})$

(ii)Subtraction: $\tilde{A} - \tilde{B} = (a_0, a_*, a^*) - (b_0, b_*, b^*)$
$= (a_0 - b_0 , \min\{a_*, b_*\}, \min\{a^*, b^*\})$

(iii)Multiplication: $\widetilde{A} \times \widetilde{B} = (a_0, a_*, a^*) \times (b_0, b_*, b^*)$
$= (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$

(iv) Division: $\widetilde{A} \div \widetilde{B} = (a_0, a_*, a^*) \div (b_0, b_*, b^*)$
$= (a_0 \div b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$

3. FUZZY TRANSPORTATION PROBLEM

3.1. Mathematical formulation of Fuzzy Transportation Problem

Consider a fuzzy transportation with m sources and n destinations with triangular fuzzy numbers. Let $a_i(a_i \geq 0)$ be the availability at source i and $b_j(b_j \geq 0)$ be the requirement at destination j. Let $\tilde{C}_{ij}$ be the unit fuzzy transportation cost from source i to destination j. Let x_{ij} denote the number of fuzzy units to be transported from source i to destination j. Now the problem is to determine a feasible way of transporting the available cost is minimized.

The Mathematical model of Fuzzy Transportation Problem is as follows:

$$Minimize \quad \tilde{Z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\, x_{ij}$$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,3, \dots m$
$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,3, \dots n$
$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \quad i = 1,2, \dots m \ and \ j = 1,2, \dots n$

and $x_{ij} \geq 0$.

This fuzzy transportation problem is explicitly represented by the following fuzzy transportation table.

Table 3.1: Fuzzy Transportation Problem

	Destination					
Sources		1	2	⋯	n	Supply
	1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	⋯	$\tilde{c}_{1n}$	a_1
	2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	⋯	$\tilde{c}_{2n}$	a_2
	⋮	⋮	⋮	⋮	⋮	⋮
	m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	⋯	$\tilde{c}_{mn}$	a_m
	Demand	b_1	b_2	⋯	b_n	

Definition 3.1. A set of non-negative allocations x_{ij} which satisfies (in the sense equivalent) the row and the column restrictions is known as fuzzy feasible solution.

Definition 3.2. A fuzzy feasible solution to a fuzzy transportation problem with m sources and n destinations is said to be a fuzzy basic feasible solution if the number of positive allocations are (m + n - 1). If the number of allocations in a fuzzy basic solution is less than (m + n - 1), it is called fuzzy degenerate basic feasible solution.

Definition 3.3. A fuzzy feasible solution is said to be fuzzy optimal solution if it minimizes the total fuzzy transportation cost.

4. ALGORITHMS

The objective of fuzzy transportation problem is, to minimize the total fuzzy transportation cost. In this paper the fuzzy transportation problem is solved by fuzzy version of Vogel's Approximation Method (FVAM) and fuzzy version of Modified Distribution Method (FMODI)

4.1. Fuzzy Version of Vogel's Approximation Algorithm (FVAM)

This algorithm is used to obtain Initial Fuzzy Basic Feasible Solution (IFBFS)

Step 1: From the Fuzzy Transportation table, determine the penalty for each row or column. The penalties are calculated for each row column by subtracting the lowest cost element in that row or column from the next cost element in the same row or column. Write down the penalties below and aside of the rows and columns respectively.

Step 2: Identify the column or row with largest fuzzy penalty. In case of tie, break the tie arbitrarily. Select a cell with minimum fuzzy cost in the selected column (or row), and assign the maximum units possible by considering the demand and supply position corresponding to the selected cell.

Step 3: Delete the column/row for which the supply and demand requirements are met.

Step 4: Continue steps 1 to 3 for the resulting fuzzy transportation table until the supply and demand of all sources and destinations have been met.

4.2. Fuzzy Version of MODI Algorithm (FMODI)

This algorithm is used to test the initial fuzzy basic feasible solution obtained is an optimal solution to Fuzzy Transportation Problem.

Step 1: Given an initial fuzzy basic feasible solution of a fuzzy transportation problem in the form of allocated and non-allocated cells of fuzzy transportation table. Assign the auxiliary variables $\tilde{u}_i$, i=1,2,3,…,m and $\tilde{v}_j$, j=1,2,3,…,n for rows and columns respectively. Compute the values of $\tilde{u}_i$ and $\tilde{v}_j$ using the relationship $\tilde{c}_{ij} = \tilde{u}_i + \tilde{v}_j$ for all i, j , for all occupied cells. Assume either $\tilde{u}_i$ or $\tilde{v}_j$ to zero which is associated with the row or column of the transportation table that contains the maximum number of allocated cells.

Step 2: For each unoccupied cell (i, j), compute the fuzzy opportunity cost $\tilde{\delta}_{ij}$, using $\tilde{\delta}_{ij} \approx \tilde{c}_{ij} - (\tilde{u}_i + \tilde{v}_j)$.

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Step 3: (i) If all $\tilde{\delta}_{ij} \succcurlyeq \tilde{0}$ then the current solution under the test is optimal.

(ii) If at least one $\tilde{\delta}_{ij} \prec \tilde{0}$ then the current solution under the test is not optimal and proceeds to the next step.

Step 4: Select an unoccupied cell (i, j) with most negative opportunity cost among all unoccupied cells.

Step 5: Draw a closed path involving horizontal and vertical lines for the unoccupied cells starting and ending at the cell obtained in step 4 and having its other corners at some allocated cells. Assign $+\theta$ and $-\theta$, alternately starting with $+\theta$ for the selected unoccupied cells.

Step 6: On the closed path, identify the corners with $-\theta$. Select the smallest allocation among the corners with $-\theta$ which indicate the number of units that can be shifted to some other unoccupied cells. Add this quantity to those corners marked with $+\theta$ and subtract this quantity to those corners marked with $-\theta$ on the closed path and check whether the number of nonnegative allocations is (m+n-1) and repeat step 1 to step 7, till we reach $\tilde{\delta}_{ij} \succcurlyeq \tilde{0}$ for all i and j.

5. NUMERICAL EXAMPLE

Consider an example given in [1], a balanced fuzzy transportation problem in which all the cost coefficients are triangular fuzzy numbers and all demands and supplies are crisp numbers.

Table 5.1: Balanced Fuzzy Transportation Problem

Source	Destination: Baghdad	Anbar	Arbil	Dohnuk	Supply
Karbala	(50,70,100)	(40,60,80)	(50,60,70)	(60,100,130)	100000
Diyala	(30,50,60)	(30,40,50)	(60,80,100)	(60,90,200)	200000
Basra	(20,25,30)	(50,60,70)	(70,90,110)	(80,100,120)	400000
Demand	200000	100000	150000	250000	700000

To apply the proposed algorithm and the fuzzy arithmetic, let us express all the triangular fuzzy numbers in the given problem based upon both location index and fuzziness index functions. That is, in the form of $\tilde{A} = (a_0, a_*, a^*)$, we have the transportation table 5.2.

In table 5.2. applying Fuzzy version of Vogel's Approximation Method (FVAM)), the initial fuzzy basic feasible solution in terms of location index and fuzziness index is shown in table 5.3.

Table 5.2: Balanced Fuzzy Transportation Problem in which all the triangular fuzzy numbers are of the form (a_0, a_*, a^*).

	Baghdad	Anbar	Arbil	Dohnuk
Karbala	$(70,20-20\alpha,30-30\alpha)$	$(60,20-20\alpha,20-20\alpha)$	$(60,10-10\alpha,10-10\alpha)$	$(100,40-40\alpha,30-30\alpha)$
Diyala	$(50,20-20\alpha,10-10\alpha)$	$((40,10-10\alpha,10-10\alpha)$	$(80,20-20\alpha,20-20\alpha)$	$(90,30-30\alpha,110-110\alpha)$
Basra	$(25, 5-5\alpha,5-5\alpha)$	$(60,10-10\alpha,10-10\alpha)$	$(90,20-20\alpha,20-20\alpha)$	$(100,20-20\alpha,20-20\alpha)$

Table 5.3: Initial Fuzzy Basic Feasible Solution

	Baghdad	Anbar	Arbil	Dohnuk

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Basra	**Diyala**	**Karbala**
$(25, 5\text{-}5\alpha,5\text{-}5\alpha)$ **200000**	$(50,20\text{-}20\alpha,10\text{-}10\alpha)$	$(70,20\text{-}20\alpha,30\text{-}30\alpha)$
$(60,10\text{-}10\alpha,10\text{-}10\alpha)$	$(40,10\text{-}10\alpha,10\text{-}10\alpha)$ **100000**	$(60,20\text{-}20\alpha,20\text{-}20\alpha)$
$(90,20\text{-}20\alpha,20\text{-}20\alpha)$ **50000**	$(80,20\text{-}20\alpha,20\text{-}20\alpha)$	$(60,10\text{-}10\alpha,10\text{-}10\alpha)$ **100000**
$(100,20\text{-}20\alpha,20\text{-}20\alpha)$ **150000**	$(90,30\text{-}30\alpha,110\text{-}110\alpha)$ **100000**	$(100,40\text{-}40\alpha,30\text{-}30\alpha)$

From table 5.3. , the initial fuzzy transportation cost is given by

$\approx$ $(60,10\text{-}10\alpha,10\text{-}10\alpha)(100000)$+ $(40,10\text{-}10\alpha,10\text{-}10\alpha)$ (100000) + $(90,30\text{-}30\alpha,110\text{-}110\alpha)$ (100000) + $(25, 5\text{-}5\alpha,5\text{-}5\alpha)$ (200000) + $(90,20\text{-}20\alpha,20\text{-}20\alpha)(50000)$ + $(100,20\text{-}20\alpha,20\text{-}20\alpha)$ (150000) $\approx$ $(4350000, 30\text{-}30\alpha,110\text{-}110\alpha)$.

In table 5.3**.** by applying the fuzzy version of Modified Distribution Method (FMODI), it can be seen that the current initial fuzzy basic feasible solution is optimal
Therefore, the corresponding fuzzy optimal transportation cost is given by,

$$Minimize\ \ \tilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\,\tilde{x}_{ij}$$
$$\approx (4350000, 30\text{-}30\alpha,110\text{-}110\alpha)\ ,$$

where $0 \le \alpha \le 1$ can be suitably chosen by the decision maker.

For αα = 0, the corresponding fuzzy optimal transportation cost is given in terms of (a_1, a_2, a_3) is

$$Minimize\ \ \tilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\,x_{ij}$$
$$= (4349970, 4350000,4350110).$$

For the same problem, Wakas S. Khalaf et.al. [1] have obtained the minimum fuzzy transportation cost as Rs. (34000000,43500000,57000000). Note that the solution obtained by our methodology is better than the solution obtained by them.

6. CONCLUSION

The transportation costs are considered as imprecise numbers described by triangular fuzzy numbers which are more realistic and general in nature. We expressed all the triangular fuzzy numbers in the given problem interms of both location index and fuzziness index functions, and obtained an optimal fuzzy feasible solution using a fuzzy version of VAM and MODI algorithms without converting them to classical transportation problems.

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